ENHANCING ELEMENTARY SCHOOL MATHEMATICS TEACHERS' MATHEMATICAL THINKING THROUGH IN-SERVICE TRAINING

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The curriculum emphasizes the development of higher order thinking skills (HOTS) and desirable dispositions while pupils learn mathematics. These are viewed as how-to-learn skills and dispositions that can empower them to learn on their own all throughout life. An important effort being done by the Department of Education (DepED) in equipping teachers to make this development happen in the classroom is by providing them in-service training. By making the teachers go through the experience of using these skills and dispositions in the context of learning mathematics lessons, they can anticipate better how their pupils might think and behave and what they can do to promote said skills and dispositions. This paper presents some examples of mathematical tasks that were used in training the teachers to enhance their mathematical thinking.

EFFORTS TO IMPROVE THINKING SKILLS

Current classroom mathematics teaching is generally procedural rather than conceptual and so does not promote understanding and problem solving (San Jose, 1998). TIMSS results revealed that Filipino students are weak at items on reasoning. These are those that require making conjectures/predictions, analysis, generalization, synthesis, evaluation, and proof. They are also not used to answering constructed-response items, particularly open-ended ones. These are items that have many different correct solutions and/or answers as well as items that require them to explain their solutions and /or answers. Multiple-choice items are commonly used in schools (Lumpas 1997). So something needs to be done to address these problems.

The elementary mathematics curriculum which has been implemented since 2002 increased the daily time allotment for the subject from 60 minutes to 80 minutes in the lower grade levels. This was to ensure that "there would be more activities that involve grouping, practical investigations, and problem solving. Pupils learn more if they have hands on or manipulative and interactive activities, learn on their own, explore, discover, generalize, and apply what they have learned in daily life." Among the curriculum's goal is for the pupils to demonstrate understanding and skill in communicating, thinking analytically and critically, and solving problems in daily life (DepED 2002, DepED Bureau of Elementary Education 2003).

Actually, the previous curriculum also advocated a classroom scenario where pupils are dynamically engaged in learning mathematics as being promoted in the current curriculum. But the increase in time allotment provided for by the current curriculum is a sincere move to provide a supportive environment for this kind of learning to take place. Projects such as the Philippines-Australian Science and Mathematics Education Project (PASMEP) funded by DepED and AusAid (1988-1993) and the Science and Mathematics Education Manpower Development Project (SMEMDP) funded by DepED and JICA (1994-1999), with NISMED as the implementing agency, greatly helped in actualising the emphasis on problem solving through their teacher training components. In particular, in SMEMDP, in-service teacher training programs were designed to model teaching mathematics through problem solving using practical work. Teaching strategies were modelled such that they offered opportunities for the teachers to experience working on open-ended tasks. These also required from them multiple solutions to problems and communication of their thinking processes in doing the mathematical activities.

TEACHING MATHEMATICS THROUGH PROBLEM SOLVING TO DEVELOP MATHEMATICAL THINKING

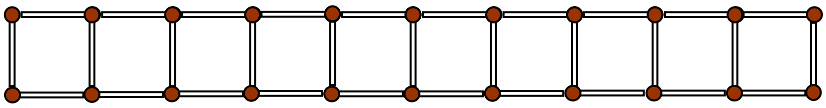
The following are examples of the activities given to the teachers. They aimed to make the teachers realize that topics in the mathematics curriculum can be used to develop thinking if they are contextualized in problem situations.

An Example of Teaching a Lesson Through Problem Solving

The context of this grade 6 - lesson was solving multi-step problems involving whole numbers. Specifically, the competencies involved were: writing an equation to solve a multi-step problem, solving a multi-step problem involving 2 or 3 operations, and describing one's answer in complete sentences.

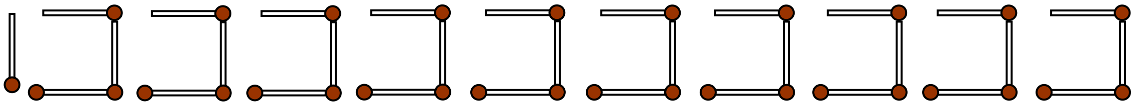
The author trained at NISMED and elsewhere, the teachers who served as sources of the data that follow. Thus, any method presented here means that there was at least one teacher who offered it. An idea suggested by a JICA mathematics expert was used as a basis for developing the lesson.

First, the teachers were asked what they see when the figure below was shown to them. Answers were: railroad, windows, chocolate bars, stairs, boxes, squares, and matchsticks.

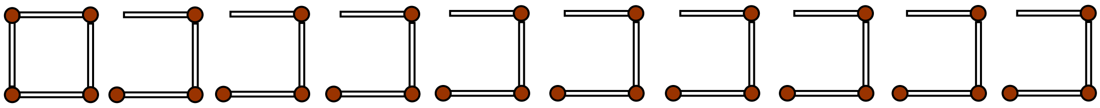


After a while, the figure was kept and the teachers were asked how many matchsticks were used to form the figure. Most correctly answered 31. They were then asked to (1) show as many different ways as they can of systematically counting the number of matchsticks, (2) explain their methods, and (3) write the number sentence for each method. Below is a summary of the participants' responses:

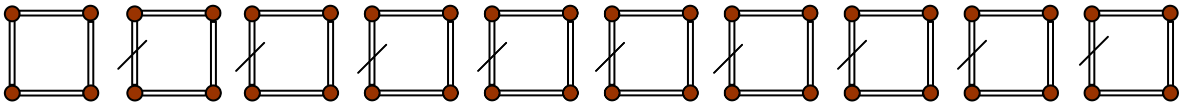
Method 1: Separate one stick first. Then only 3 sticks are needed to form each square as drawn. Then the total number of matchsticks is 1 + (10 x 3) = 31.



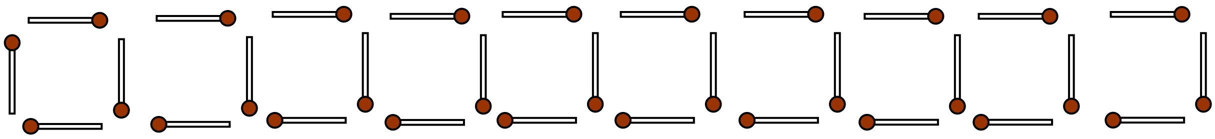
Method 2: Take one square first. This has 4 matchsticks. Then the next squares would need only 3 sticks each to be formed. For 10 squares, the total number of matchsticks is 4 + (10-1) x 3 = 31.



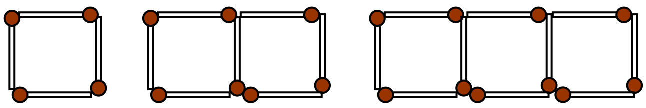
Method 3: Think first of the squares as being separate from each other. So there are 10 x 4 = 40 sticks. However, in reality the squares share a common side. So they share (10-1) sticks and these should be subtracted from the total. The number sentence therefore is (10 x 4) – (10 – 1) = 31.



Method 4: The squares consist of horizontal sides and vertical sides. There are 2 rows of horizontal sides, the top and the bottom, and each has 10 sticks each. There are 1 + 10 vertical sides. So the total number of sticks is (2 x 10) + (1 + 10) = 31.



Method 5: Consider increasing number of squares, from 1 square, 2 squares, 3 squares, and so on. Then find the total number of sticks for each. See if there is a pattern in the numbers generated. So, it will be 4, 7, 10, 13, 17, … In table form, it will be:

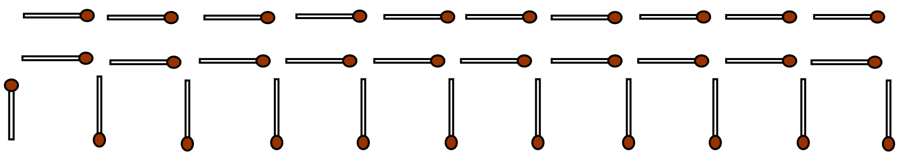


. . .

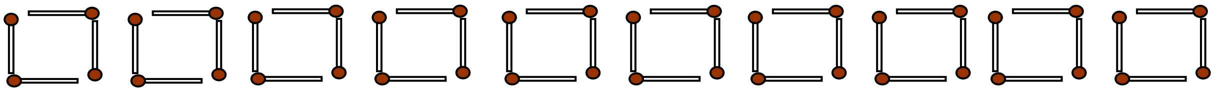
Number of squares	1	2	3	4
Number of matchsticks	4	7	10	13

The relationship appears to be (3 x number of squares) + 1. Then for 10 squares, the number of matchsticks is (3 x 10) + 1 = 31.

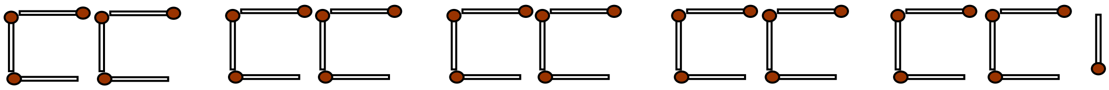
Method 6: Instead of counting the individual horizontal sticks from left to right at the top and then at the bottom, pairs of horizontal sticks, one at the top and the other at the bottom are counted from left to right. And for the 10 squares, this is (10 x 2) + (10 + 1) = 31.



Method 7: For each square, count the pair of vertical and horizontal sticks on the lower left corner, then the pair of horizontal and vertical sticks in the upper right corner, as if the squares are separated. The total number of sticks would be 10 x (2 x 2). But adjoining squares have a common side so (10 – 1) sticks should be subtracted from this number. So the number of matchsticks used is 10 x (2 x 2) – (10 – 1) = 31.



Method 8: Consider two consecutive squares such that their sticks form a "chair" as shown. There will be (5 x 6) + 1= 31 sticks in all.



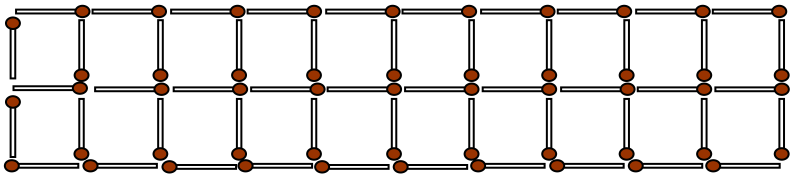
The different methods showed that a single mathematical idea could be represented in different forms – using drawings, words, and symbols (NCTM 2000). For all the methods except Method 5, it was clear that every number and every operation could be associated with a physical meaning in the figure. If the answer was obtained using Method 5, then the number sentence was the same as that in Method 1 and this could be explained using the explanation in Method 1. The teachers appreciated this multi-representation of a mathematical idea very much. Asking students for different representations and identifying the relationships among them is something that they do not do in their classes.

The teachers also appreciated that although the methods are all different, the answer was always the same. They also realized that each method represented a different way of systematic counting or a different way of thinking. When asked what was the importance of exploring different methods of solving a problem, the teachers said that pupils think differently and how they think should be respected. Some said that using other methods could help one check the correctness of the answer obtained using a certain method. The underlying reasoning for all the methods except for Method 5 was obvious because of the visual representation given by the drawing. But Method 1 could provide the explanation for Method 5.

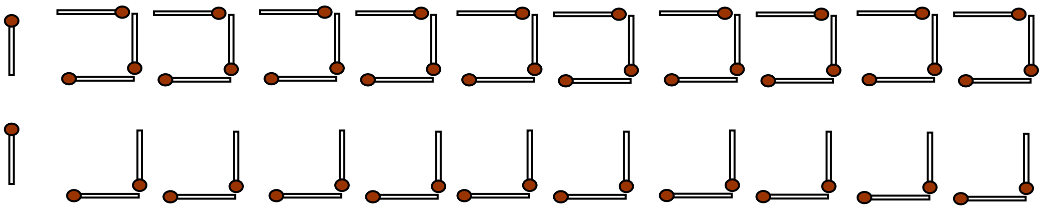
The teachers were then told that there was going to be a contest. They were to tell the number of matchsticks used to form different numbers of squares. Bigger numbers were given. There were those who consistently gave the correct answer first, ahead of all the rest. When asked what method they used, they said that they used Method 1. Through this, the teachers were led to realize that although there were several different correct methods, there was a method that might be better than the others. To most, Method 1 was the easiest to visualize so they chose it. If efficiency were the criteria, then Method 1 would again be the winner because it used only 2 operations unlike the others that used more. Incidentally, judging that Method 1 was the best based on how quickly it could enable one to get the answer was an example of critical thinking. Critical thinking evaluates accuracy, validity, efficiency, and the like. However, the choice as to what method was good depended on the person. According to some teachers, pupils in the lower elementary grades would most likely use the method of considering the kind of sticks, whether vertical or horizontal. Through the contest, the teachers saw which numbers changed and which remained in their number sentence when they predicted the number of matchsticks used for varying numbers of squares. From these predictions, they were able to generalize their number sentence. They also realized that some methods did not work for all numbers such as method 8. When the number of squares was odd, the number sentence should be changed.

A modification of the problem was presented in order to determine if the teachers could apply the methods that they have learned to a slightly different figure. It was invoking what Polya (1957) suggested that one should ask, to solve a problem: "Have I solved a similar problem before?" Also, this was to show the teachers that from a single original problem, other problems could be formulated by changing certain conditions.

What if there are now 2 rows of squares. How many sticks are there?

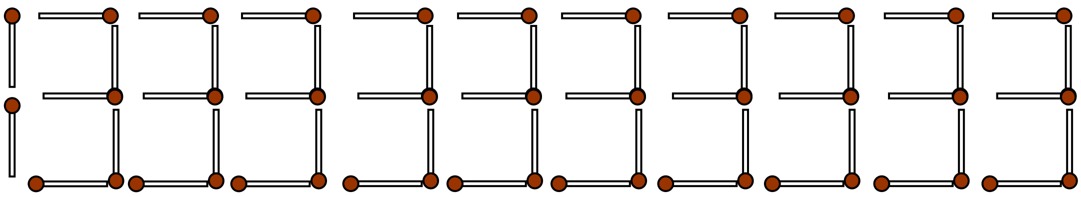


Some teachers applied the previous methods separately to each row of squares making some modifications, as a result. For example, Method 1 was applied as follows.



And the number sentence that they gave was: [1 + (10 x 3)] + [1 + (10 x 2)] = 52.

Others applied Method 1 to the entire figure viewing it as one as shown below.



Their number sentence was 2 + (10 x 5) = 52.

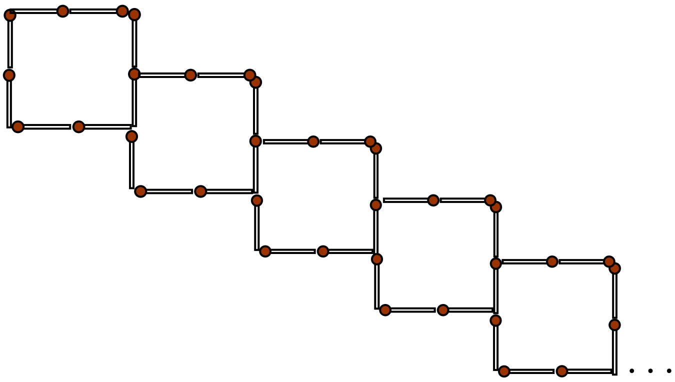
However, when they tried to generalize, they considered the total number of squares for both rows and the generalization they made was $\left(\frac{n}{\mathbf{2}} \text{ x } \mathbf{5}\right) + \mathbf{2}$ where n represented the total number of squares in the 2 rows. Later, they realized that since the number of squares was the same for each row, they changed their definition of n to be the number of squares per row and gave the expression (n x 5) + 2 for the number of sticks used.

In the same way, other methods were used similarly.

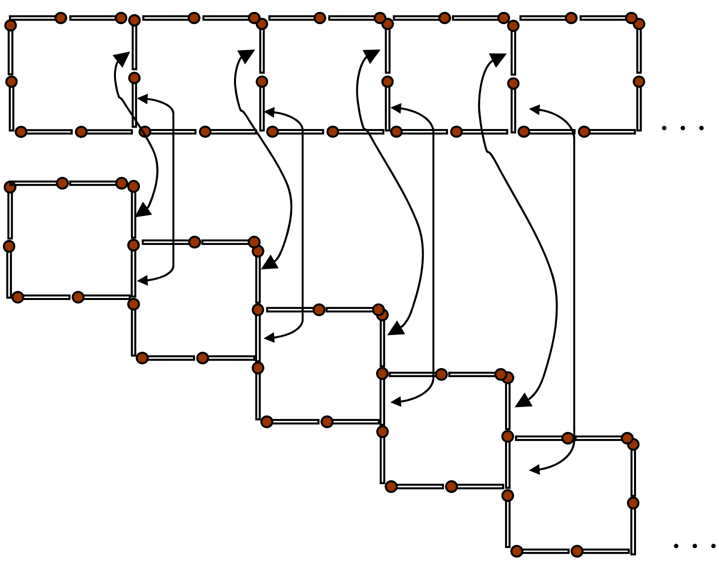
The teachers were asked to formulate related questions based on the original one. These were what they asked: “What if we have triangles instead of squares?” “What if we are given the total number of sticks and we are asked to find the number of squares that may be formed as arranged in the original figure?”

Other modifications of the original problem were: “Is it possible to have a total number of sticks equal to 62, that is, double 31? If yes, how is this achieved? If no, why not?” This previous question was linked to the next question when the teachers could not give the correct answer. “What if the lengths of the sides of each square are doubled – that is, there are 2 sticks on each side of every square?” The teachers were sure that all their earlier methods except for Method 8 still apply given any number of squares, which is at least 1.

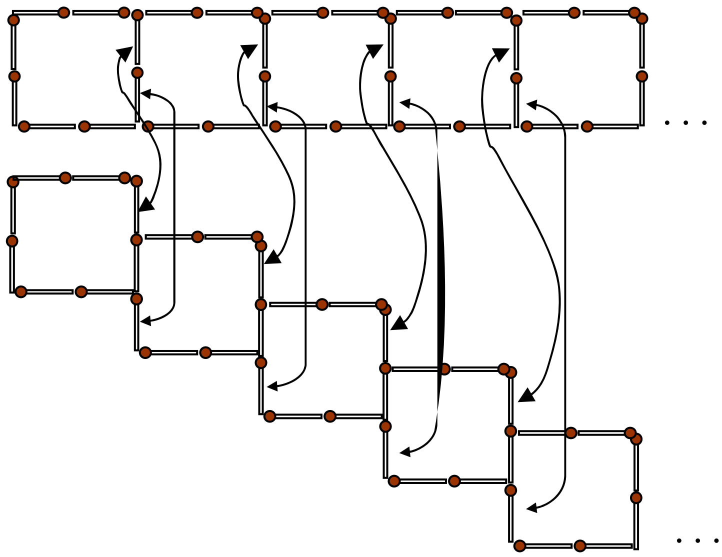
Afterwards, combinations of modifications on the original problem were made. The teachers were asked, “What if instead of having the squares arranged in a row, they are arranged diagonally like a ladder as shown and the lengths of the sides are doubled?”



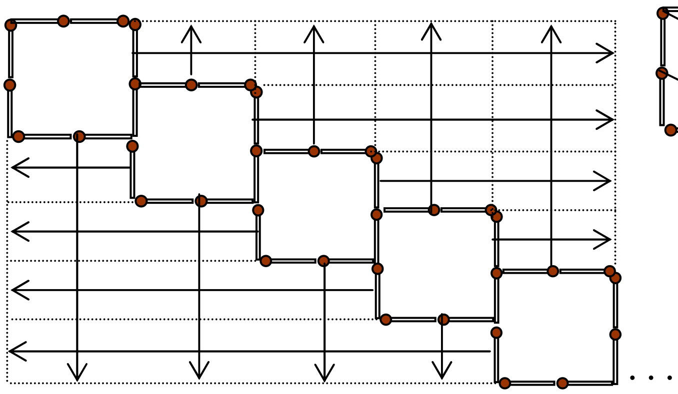
The teachers realized that with this new arrangement, there was an additional stick at every “joint” as shown below. For any given number of squares, which is at least 1, all the previous methods except for Method 8 still worked. However, the teachers needed to consider the additional stick at every joint. For example, using Method 1 gives 2 + (10 x 6) + (10 – 1) = 71 sticks when there are 10 squares.



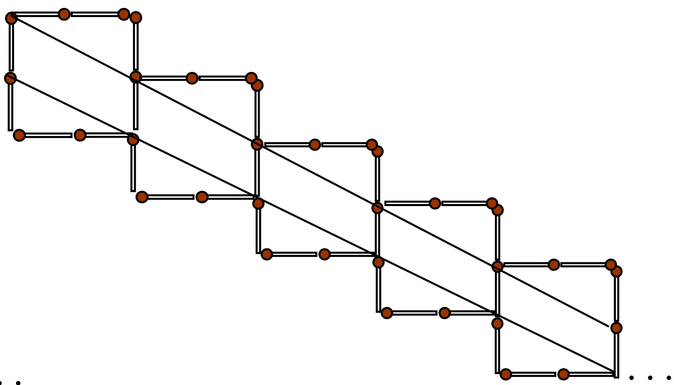
Still another extension problem given to the teachers, was: "What if the conditions are as above (i. e. the squares are arranged diagonally and the lengths of the sides are doubled) but this time it is the perimeter that is asked for instead of the total number of sticks? Each stick is 1 unit long." They realized that this problem is related to finding the total number of sticks used when the squares having 2 sticks on each side are placed side by side in a row. So Methods 1 to 7 can be used to get the answer with the unit for perimeter considered.



Interestingly, they generated other methods with the corresponding number sentences for 10 squares, as shown below.



2 x (10 x 2) + (10 + 1) x 2 = 62 sticks



2 x (10 x 3) + 2 = 62 sticks

Incidentally, the methods exemplified creative thinking because they were different from all the other methods. Creative thinking results to an output that is new, different, and the like.

All the competencies stated at the beginning were all addressed in the activity in the lesson. Moreover, different thinking skills were used in the process. At this stage, the framework of core thinking skills by Marzano, Brandt, Hughes, et al (1988) was discussed with the teachers. They were asked to cite specific instances in the lesson where they experienced using the thinking skills listed below:

- Focussing skills: defining problems, setting goals
- Information-gathering skills: observing, formulating questions
- Remembering skills: encoding, recalling
- Organizing skills: comparing, classifying, ordering, representing
- Analysing skills: identifying attributes and components, identifying relationships and patterns, identifying errors
- Generating skills: inferring, predicting, elaborating
- Integrating skills: summarizing, restructuring
- Evaluating skills: establishing criteria, verifying.

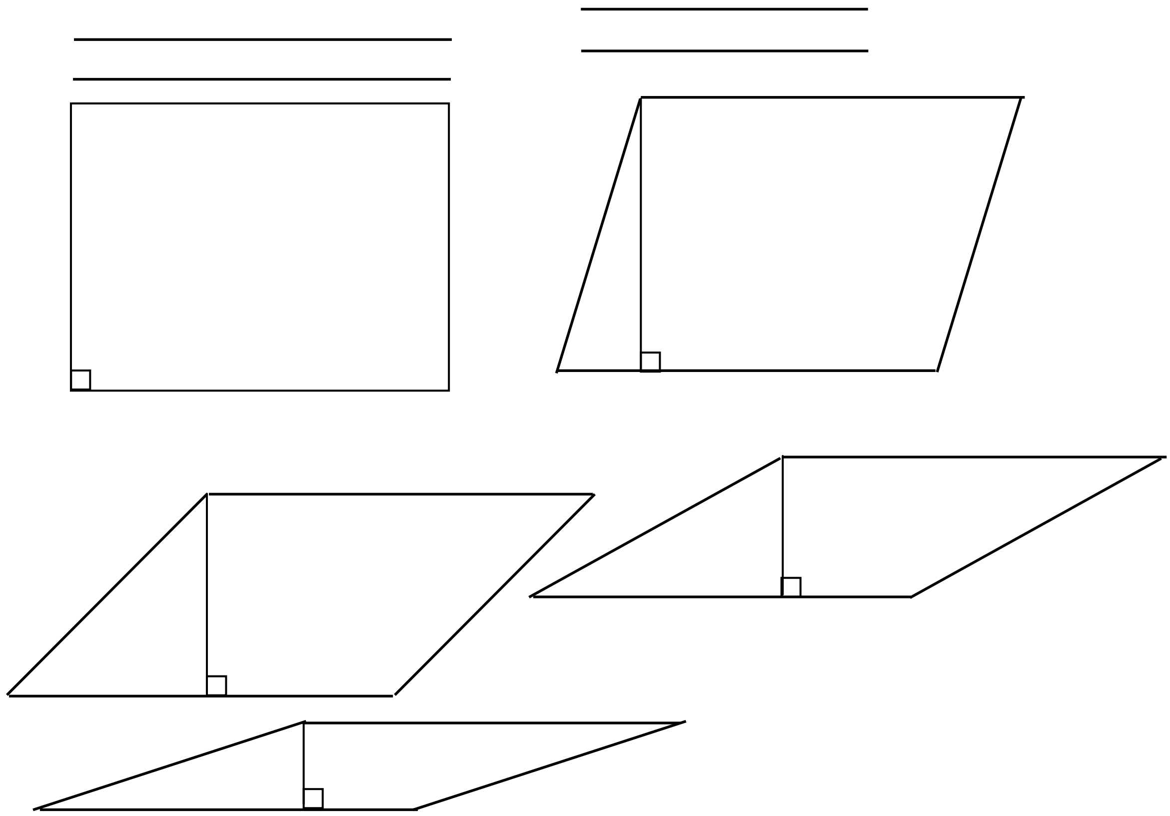
The teachers appreciated how much thinking they used while doing mathematics. They also liked how, by asking related questions, a simple problem was made more complex. They were amazed that the methods that worked for the simple cases were also the basis of the methods for the more complex ones.

Examples that Provoke Thinking through Surprise or Contradiction

Mathematical thinking may be triggered by a gap, a contradiction, or a surprise (Mason, Burton & Stacey 1990, Romberg 1994). Considering that Filipino students find it difficult to consider two or more attributes simultaneously, particularly in measurement (Lumpas 1997), the examples given here are related to measurement.

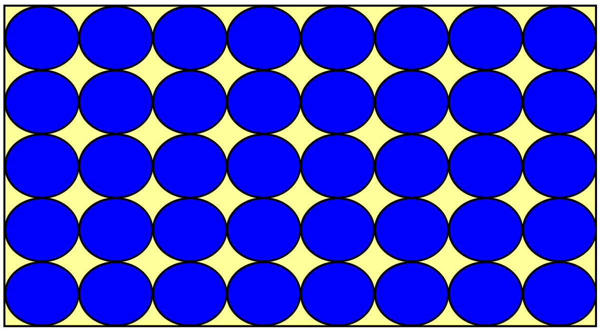
Example 1: The teachers were given two identical sheets of rectangular paper with the same shape and size. They were asked what they observed about them and they were able to answer correctly. They were then asked to form two different open cylinders by turning each sheet and taping them only along one pair of opposite sides so that there were no overlaps. They were able to form one slim but tall cylinder and one stout but short cylinder. They were asked to predict what relationship existed between their volumes. Some said that the volumes were the same since the areas of the sheets where they came from were the same. Others said that they were different and that the stouter one had a bigger volume while others said that the taller one had a bigger volume. By actually filling one cylinder with sand and transferring the contents into the other, they discovered that the stouter one in the case given, had a bigger volume. Some teachers tried computing the volume of each cylinder. They considered the length of one sheet as the circumference of the base of one cylinder. Then they considered the width of the other sheet as the circumference of the base of the other cylinder. The results confirmed the relationship. So they learned that if two objects are the same or equal in one attribute, they are not necessarily the same or equal in another attribute. They wanted to explore what the relationship between the length and the width of the rectangular sheet must be so that the volumes of the two cylinders will be equal, if possible. Likewise, they wanted to find out the conditions that the length and width of the rectangular sheet must satisfy, so that the taller cylinder will have the bigger volume, if also possible.

Example 2: The teachers were given sets of 4 strips as shown below. In all the sets, the lengths of strips are of two kinds only. Then they were asked, “If you were to form different parallelograms, what would be true about them?” And they said that they have the same perimeter. Then they were asked to actually make the parallelograms and observe other relationships. Some observed that the areas of the parallelograms became smaller as a pair of opposite sides became closer to each other. They reasoned that although the base remained the same for all the parallelograms, the height or altitude became smaller.



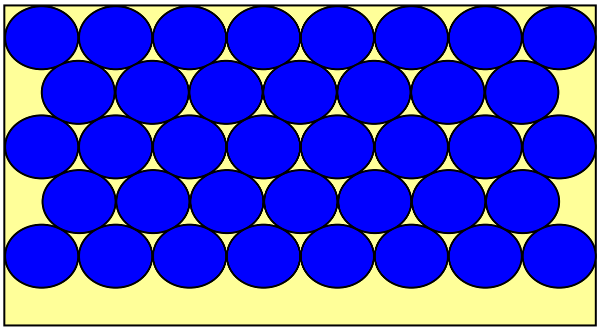
Example 3: This problem was given: Each circular chip has a diameter of one unit. The mat is 8 units by 5 units. What is the greatest number of chips that may be exactly contained on the mat such that there are no overlaps?

All teachers readily answered 40 chips. They explained that they sort of considered the chips as squares. Then they were given more than 40 chips and a mat to verify their answer. They arranged the chips as shown below. They realized that there were spaces that were not covered but they said that no extra chip could fit there.



5 x 8 = 40 chips

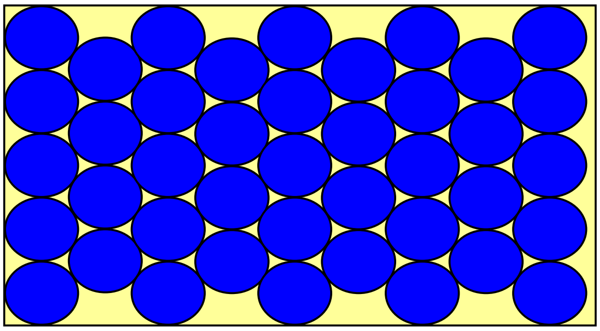
When asked if there were other ways of arranging the chips, it took them a long time to think about it. In fact, because, they observed that more spaces were created by a new arrangement, some of them did not continue to fill the mat with chips. But some patiently carried on and obtained the two other arrangements below. Eventually, they realized that by making the spaces between the chips smaller, they could have more spaces to accommodate possibly an additional chip, in this case, only one more. And this was achieved by a new arrangement of the chips. This experience made the teachers realize the importance of being able to think flexibly. They wanted to explore in what other instances or dimensions of rectangles can a similar case be possible.



3 x 8 = 24

2 x 7 = 14

38 chips



5 x 5 = 25

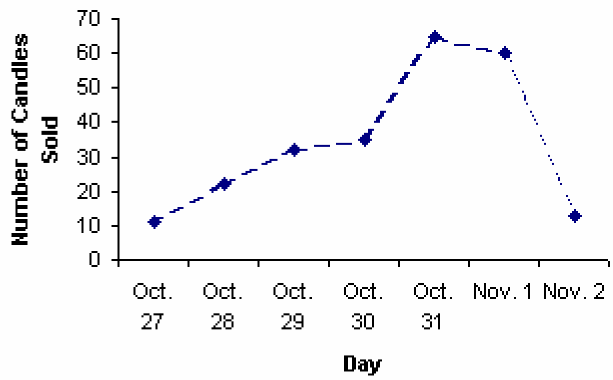
4 x 4 = 16

41 chips

Examples of Open-ended Assessment Items

Following are examples of open-ended assessment items that were also given to the teachers.

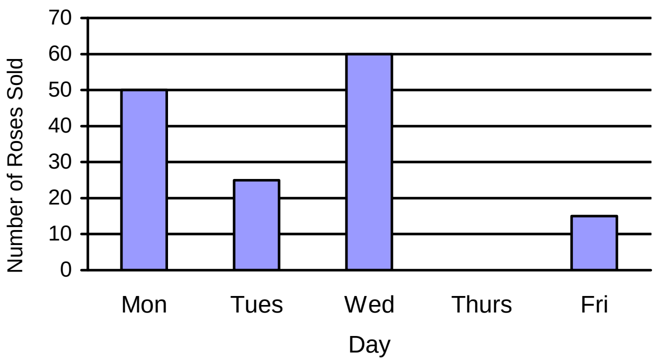
1. For items 1 and 2, give your observations, interpretations and inferences based on the graphs shown.



One Week Sale of Candles at Rosie's Store

The usual questions asked related to graphs such as this, are: "On what date was the sale of candles the greatest?" "How much was the sale?" But better than these, was to exploit the context depicted by the graph. In the Philippines, it is a custom to remember dead people on November 1 and 2 as All Saints Day and All Souls Day, by lighting candles and bringing flowers to the cemetery. The teachers were asked to tell what they observed, what their interpretation was for what they observed, and what inferences they could make based on their interpretation, related to the given graph. One observation was that the highest point was on October 31. They interpreted it as: the sale of candles was greatest on October 31. They inferred that this was because people had received their salaries on October 30 and so they had money to buy candles already. Others inferred that October 31 was the day before All Saints Day and so, most of them bought candles only then. The teachers realized that there could be other inferences.

2.



Sale of Roses at Mary's Flower Shop

For this next graph, the teachers observed that there was no bar for Thursday. They interpreted it as no roses were sold on said day. Their inferences were: (1) The shop was closed. (2) The shop was open but Mary had no rose to sell although some wanted to buy roses. Perhaps, no roses were delivered to her store that day. (3) The shop was open and Mary had roses to sell but no one bought roses.

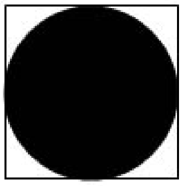
These examples on graphs showed how thinking skills like observing, interpreting, and inferring differed from, but were related to each other and how these items were open-ended.

3. The squares in the three figures below have the same area.

Figure 1

Figure 2

Figure 3

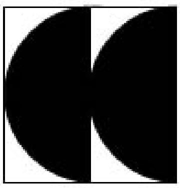


Draw 3 more figures so that they also have the same characteristics common to the first three.

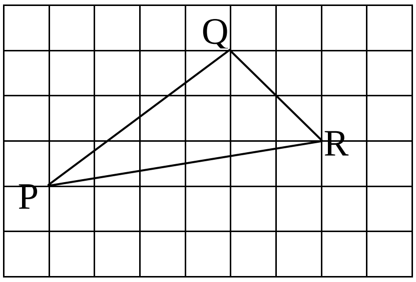
Some of the answers given by the teachers were:

Figure 4

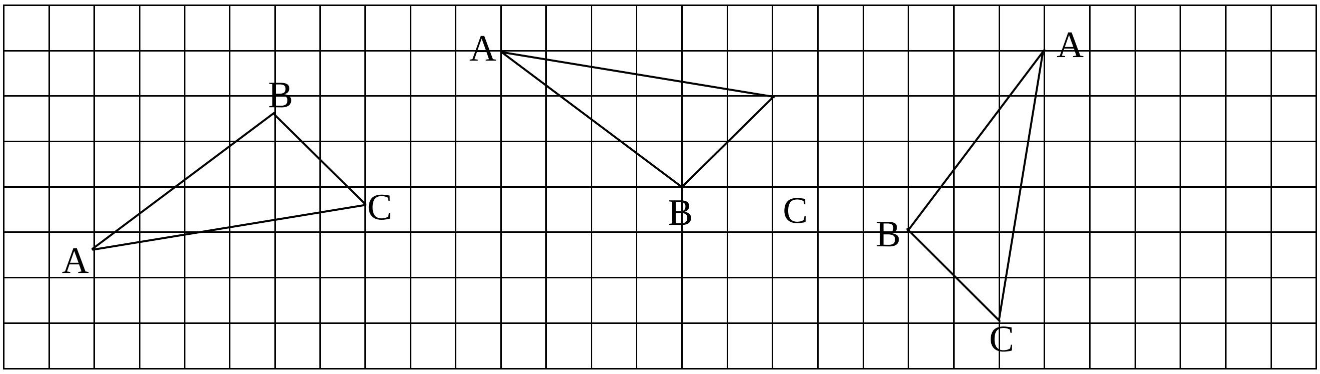
Figure 5



4. Draw another triangle, call it triangle ABC, that is congruent to ΔPQR.



The teachers flipped, turned, slid the figure, or used the idea of slope to draw the required triangle, as shown below.



WHAT ELSE NEEDS TO BE DONE?

Feedbacks from the teachers after the training showed that they learned and appreciated the learner-centred strategies and high level questions that were used. However, they commented that they were not sure if they could implement these in their classes because pupils are not used to thinking on their own. Thinking needs time and they worry that they may not be able to cover the required content. With the addition of time and encouragement to use interactive activities provided by the curriculum, the teachers should be more courageous to try the approaches they have experienced. This is where lesson study may be of great help, planning lessons that use among others effective questioning techniques and can cover several competencies, using a single problem in order to address both the content and the thinking skills needed to learn it.

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